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Mass matrix *Ansatz* and lepton flavor violation in the THDM-III

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Abstract

Predictive Higgs-fermion couplings can be obtained when a specific texture for the fermion mass matrices is included in the general two-Higgs doublet model. We derive the form of these couplings in the charged lepton sector using a Hermitian mass matrix *Ansatz* with four-texture zeros. The presence of unconstrained phases in the vertices $\phi_i l_i l_j$ modifies the pattern of flavor-violating Higgs interactions. Bounds on the model parameters are obtained from present limits on rare lepton flavor violating processes, which could be extended further by the search for the decay $\tau \to \mu\mu\mu$ and $\mu - e$ conversion at future experiments. The signal from Higgs boson decays $\phi_i \to \tau\mu$ could be searched at the large hadron collider (LHC), while $e - \mu$ transitions could produce a detectable signal at a future $e\mu$-collider, through the reaction $e^+\mu^- \to h^0 \to \tau^+\tau^-$.

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1 Introduction.

After many years of success of the Standard Model (SM), the Higgs mechanism is still the least tested sector, and the problem of electroweak symmetry breaking (EWSB) remmains almost as open as ever. However, the analysis of raditive corrections within the SM [1], points towards the existence of a light Higgs boson, which could be detected in the early stages of LHC [2]. On the other hand, the SM is often considered as an effective theory, valid up to an energy scale of $O(TeV)$, and eventually it will be replaced by a more fundamental theory, which will explain, among other things, the physics behind EWSB and perhaps even the origin of flavor. Several examples of candidate theories, which range from supersymmetry [3] to deconstruction [4], include a Higgs sector with two scalar doublets, which has a rich structure and predicts interesting phenomenology [5]. The general two-higgs doublet model (THDM) has a potential problem with flavor changing neutral currents (FCNC) mediated by the Higgs bosons, which arises when each quark type (u and d) is allowed to couple to both Higgs doublets, and FCNC could be induced at large rates that may jeopardize the model. The possible solutions to this problem of the THDM involve an assumption about the Yukawa structure of the model. To discuss them it is convenient to refer to the Yukawa lagrangian, which is written for the quarks fields as follows:

$$\mathcal{L}_Y = Y_1^u \overline{Q}_L \Phi_1 u_R + Y_2^u \overline{Q}_L \Phi_2 u_R + Y_1^d \overline{Q}_L \Phi_1 d_R + Y_2^d \overline{Q}_L \Phi_2 d_R \tag{1}$$

where $\Phi_{1,2} = (\phi^+_{1,2}, \phi^0_{1,2})^T$ denote the Higgs doublets. The specific choices for the Yukawa matrices $Y^q_{1,2}$ $(q = u, d)$ define the versions of the THDM known as I, II or III, which involve the following mechanisms, that are aimed either to eliminate the otherwise unbearable FCNC problem or at least to keep it under control, namely:

1. *DISCRETE SYMMETRIES.* A discrete symmetry can be invoked to allow a given fermion type (u or d-quarks for instance) to couple to a single Higgs doublet, and in such case FCNC's are absent at tree-level. In particular, when a single Higgs field gives masses to both types of quarks (either $Y_1^u = Y_1^d = 0$ or $Y_2^u = Y_2^d = 0$), the resulting model is referred as THDM-I. On the other hand, when each type of quark couples to a different Higgs doublet (either $Y_1^u = Y_2^d = 0$ or $Y_2^u = Y_1^d = 0$), the model is known as the THDM-II. This THDM-II pattern is highly motivated because it arises at tree-level in the minimal SUSY extension for the SM (MSSM) [5].

2. *RADIATIVE SUPRESSION.* When each fermion type couples to both Higgs doublets, FCNC could be kept under control if there exists a hierarchy between $Y_1^{u,d}$ and $Y_2^{u,d}$. Namely, a given set of Yukawa matrices is present at tree-level, but the other ones arise only as a radiative effect. This occurs for instance in the MSSM, where the type-II THDM structure is not protected by any symmetry, and is transformed into a type-III THDM (see bellow), through the loop effects of sfermions and gauginos. Namely, the Yukawa couplings that are already present at tree-level

in the MSSM (Y_1^d, Y_2^u) receive radiative corrections, while the terms (Y_2^d, Y_1^u) are induced at one-loop level.

In particular, when the "seesaw" mechanism [6] is implemented in the MSSM to explain the observed neutrino masses [7, 8], lepton flavor violation (LFV) appears naturally in the right-handed neutrino sector, which is then communicated to the sleptons and from there to the charged leptons and Higgs sector. These corrections allow the neutral Higgs bosons to mediate LFV's, in particular it was found that the (Higgs-mediated) tau decay $\tau \to 3\mu$ [9] as well as the (real) Higgs decay $H \to \tau\mu$ [10], can enter into possible detection domain. Similar effects are known to arise in the quark sector, for instance $B \to \mu\mu$ can reach branching fractions at large $\tan\beta$, that can be probed at Run II of the Tevatron [11, 12].

3. *FLAVOR SYMMETRIES.* Suppression for FCNC can also be achived when a certain form of the Yukawa matrices that reproduce the observed fermion masses and mixing angles is implemented in the model, which is then named as THDM-III. This could be done either by implementing the Frogart-Nielsen mechanism to generate the fermion mass hierarchies [13], or by studying a certain *Ansatz* for the fermion mass matrices [14]. The first proposal for the Higgs couplings along these lines was posed in [15, 16], it was based on the six-texture form of the mass matrices, namely:

$$M_l = \begin{pmatrix} 0 & C_l & 0 \\ C_l^* & 0 & B_l \\ 0 & B_l^* & A_l \end{pmatrix}.$$

Then, by assuming that each Yukawa matrix $Y_{1,2}^q$ has the same hierarchy, one finds: $A_l \simeq m_3$, $B_l \simeq \sqrt{m_2 m_3}$ and $C_l \simeq \sqrt{m_1 m_2}$. Then, the Higgs-fermion couplings obey the following pattern: $Hf_if_j \sim \sqrt{m_i m_j}/m_W$, which is known as the Cheng-Sher *Ansatz*. This brings under control the FCNC problem, and it has been extensively studied in the literature to search for flavor-violating signals in the Higgs sector [17].

In this paper we are interested in studying the flavor symmetry option. However, the six-texture *Ansatz* seems disfavored by current data on the CKM mixing angles. More recently, mass matrices with four-texture *Ansatz* have been considered, and are found in better agreement with the observed data [18]. It is interesting then to investigate how the Cheng-Sher form of the Higgs-fermion couplings, gets modified when one replaces the six-texture matrices by the four-texture *Ansatz*. This paper is aimed precisely to study this question; we want to derive the form of the Higgs-fermion couplings and to discuss how and when the resulting predictions could be tested, both in rare tau decays and in the phenomenology of the Higgs bosons [10]. Unlike previous studies, we keep in our analysis the effect of the complex phases, which modify the FCNC Higgs couplings.

The organization of the paper goes as follows: In section 2, we discuss the lagrangian for the THDM with the four-texture form for the mass matrices, and present the results

for the Higgs-fermion vertices in the charged lepton sector. Then, in section 3 we study the constraints impossed on the parameters of the model from low energy LFV processes. In section 4 we discuss predictions of the model for tau and Higgs decays, including the capabilities of future hadron and $e\mu$-colliders to probe this phenomena. Finally, section 5 contains our conclusions.

2 The THDM-III with four-texture mass matrices

The Yukawa lagrangian of the THDM-III for the lepton sector is given by:

$$\mathcal{L}_Y^l = Y_{1ij}^l \overline{L_i} \Phi_1 l_{Rj} + Y_{2ij}^l \overline{L_i} \Phi_2 l_{Rj}. \tag{2}$$

After SSB the charged lepton mass matrix is given by,

$$M_l = \frac{1}{\sqrt{2}}(v_1 Y_1^l + v_2 Y_2^l), \tag{3}$$

We shall asuume that both Yukawa matrices Y_1^l and Y_2^l have the four-texture form and Hermitic; following the conventions of [18], the lepton mass matrix is then written as:

$$M_l = \begin{pmatrix} 0 & C_l & 0 \\ C_l^* & \tilde{B}_l & B_l \\ 0 & B_l^* & A_l \end{pmatrix}.$$

when $\tilde{B}_l \to 0$ one recovers the six-texture form. We also consider the hierarchy: $\mid A_l \mid \gg \mid \tilde{B}_l \mid, \mid B_l \mid, \mid C_l \mid$, which is supported by the observed fermion masses in the SM.

Because of the hermicity condition, both $\tilde{B}_l$ and A_l are real parameters, while the phases of C_l and B_l, $\Phi_{B,C}$, can be removed from the mass matrix M_l by defining: $M_l = P^\dagger \tilde{M} P$, where $P = diag[1, e^{i\Phi_C}, e^{i(\Phi_B+\Phi_C)}]$, and the mass matrix $\tilde{M}_l$ includes only the real parts of M_l. The diagonalization of $\tilde{M}$ is then obtained by an orthogonal matrix O, such that the diagonal mass matrix is: $\overline{M}_l = O^T \tilde{M}_l O$.

The lagrangian (2) can be expanded in terms of the mass-eigenstates for the neutral (h^0, H^0, A^0) and charged Higgs bosons $(H^\pm)$. The interactions of the neutral Higgs bosons are given by,

$$\begin{aligned} \mathcal{L}_Y^l &= \frac{g}{2}\left(\frac{m_i}{m_W}\right)\bar{l}_i\left[\frac{\cos\alpha}{\cos\beta}\delta_{ij} + \frac{\sqrt{2}\,\sin(\alpha-\beta)}{g\,\cos\beta}\left(\frac{m_W}{m_i}\right)\tilde{Y}_{2ij}^l\right] l_j H^0 \\ &+ \frac{g}{2}\left(\frac{m_i}{m_W}\right)\bar{l}_i\left[-\frac{\sin\alpha}{\cos\beta}\delta_{ij} + \frac{\sqrt{2}\,\cos(\alpha-\beta)}{g\,\cos\beta}\left(\frac{m_W}{m_i}\right)\tilde{Y}_{2ij}^l\right] l_j h^0 \\ &+ \frac{ig}{2}\left(\frac{m_i}{m_W}\right)\bar{l}_i\left[-\tan\beta\delta_{ij} + \frac{\sqrt{2}}{g\,\cos\beta}\left(\frac{m_W}{m_i}\right)\tilde{Y}_{2ij}^l\right]\gamma^5 l_j A^{0.} \end{aligned} \tag{4}$$

The first term, proportional to δij corresponds to the modification of the THDM-II over the SM result, while the term proportional to $\tilde{Y}_2^l$ denotes the new contribution from THDM-III. Thus, the fermion-Higgs couplings respect CP-invariance, despite the fact that the Yukawa matrices include complex phases; this follows because of the Hermiticity conditions impossed on both Y_1^l and Y_2^l.

The corrections to the lepton flavor conserving (LFC) and flavor-violating (LFV) couplings, depend on the rotated matrix: $\tilde{Y}_2^l = O^T P Y_2^l P^\dagger O$. We shall evaluate $\tilde{Y}_2^l$, by assuming that Y_2^l has a four-texture form, namely:

$$Y_2^l = \begin{pmatrix} 0 & C_2 & 0 \\ C_2^* & \tilde{B}_2 & B_2 \\ 0 & B_2^* & A_2 \end{pmatrix}, \qquad \mid A_2 \mid \gg \mid \tilde{B}_2 \mid, \mid B_2 \mid, \mid C_2 \mid . \tag{5}$$

The matrix that diagonalizes the real matrix $\tilde{M}_l$ with the four-texture form, is given by:

$$O = \begin{pmatrix} \sqrt{\frac{\lambda_2\lambda_3(A-\lambda_1)}{A(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)}} & \eta\sqrt{\frac{\lambda_1\lambda_3(\lambda_2-A)}{A(\lambda_2-\lambda_1)(\lambda_3-\lambda_2)}} & \sqrt{\frac{\lambda_1\lambda_2(A-\lambda_3)}{A(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}} \\ -\eta\sqrt{\frac{\lambda_1(\lambda_1-A)}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)}} & \sqrt{\frac{\lambda_2(A-\lambda_2)}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_2)}} & \sqrt{\frac{\lambda_3(\lambda_3-A)}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}} \\ \eta\sqrt{\frac{\lambda_1(A-\lambda_2)(A-\lambda_3)}{A(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)}} & -\sqrt{\frac{\lambda_2(A-\lambda_1)(\lambda_3-A)}{A(\lambda_2-\lambda_1)(\lambda_3-\lambda_2)}} & \sqrt{\frac{\lambda_3(A-\lambda_1)(A-\lambda_2)}{A(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}} \end{pmatrix},$$

where $m_e = m_1 = \mid \lambda_1 \mid, m_\mu = m_2 = \mid \lambda_2 \mid, m_\tau = m_3 = \mid \lambda_3 \mid, \eta = \lambda_2/m_2$

Then the rotated form $\tilde{Y}_2^l$ has the general form,

$$\begin{aligned} \tilde{Y}_2^l &= O^T P Y_2^l P^\dagger O \\ &= \begin{pmatrix} \tilde{Y}_{211}^l & \tilde{Y}_{212}^l & \tilde{Y}_{213}^l \\ \tilde{Y}_{221}^l & \tilde{Y}_{222}^l & \tilde{Y}_{223}^l \\ \tilde{Y}_{231}^l & \tilde{Y}_{232}^l & \tilde{Y}_{233}^l \end{pmatrix}. \end{aligned} \tag{6}$$

However, the full expressions for the resulting elements have a complicated form, as it can be appreciated, for instance, by looking at the element $(\tilde{Y}_2^l)_{22}$, which is displayed here:

$$(\tilde{Y}_2)_{22}^l = \eta[C_2^* e^{i\Phi_C} + C_2 e^{-i\Phi_C}]\frac{(A-\lambda_2)}{m_3-\lambda_2}\sqrt{\frac{m_1 m_3}{A m_2}} + \tilde{B}_2\frac{A-\lambda_2}{m_3-\lambda_2} \tag{7}$$

$$+A_2\frac{A-\lambda_2}{m_3-\lambda_2} - [B_2^* e^{i\Phi_B} + B_2 e^{-i\Phi_B}]\sqrt{\frac{(A-\lambda_2)(m_3-A)}{m_3-\lambda_2}} \tag{8}$$

where we have taken the limits: $|A|, m_\tau, m_\mu \gg m_e$. The free-parameters are: $\tilde{B}_2, B_2, A_2, A$.

To derive a better suited approximation, we shall consider the elements of the Yukawa matrix Y_2^l as having the same hierarchy as the full mass matrix, namely:

$$
\begin{aligned}
C_2 &= c_2\sqrt{\frac{m_1 m_2 m_3}{A}} && (9)\\
B_2 &= b_2\sqrt{(A-\lambda_2)(m_3-A)} && (10)\\
\tilde{B}_2 &= \tilde{b}_2(m_3-A+\lambda_2) && (11)\\
A_2 &= a_2 A. && (12)
\end{aligned}
$$

Then, in order to keep the same hierarchy for the elements of the mass matrix, we find that A must fall within the interval $(m_3 - m_2) \le A \le m_3$. Thus, we propose the following relation for A:

$$A = m_3(1-\beta z), \tag{13}$$

where $z = m_2/m_3 \ll 1$ and $0 \le \beta \le 1$.

Then, we introduce the matrix $\tilde{\chi}$ as follows:

$$
\begin{aligned}
\left(\tilde{Y}_2^l\right)_{ij} &= \frac{\sqrt{m_i m_j}}{v}\,\tilde{\chi}_{ij}\\
&= \frac{\sqrt{m_i m_j}}{v}\,\chi_{ij}\, e^{\vartheta_{ij}} && (14)
\end{aligned}
$$

which differs from the usual Cheng-Sher *Ansatz* not only because of the appearence of the complex phases, but also in the form of the real parts $\chi_{ij} = |\tilde{\chi}_{ij}|$.

Expanding in powers of z, one finds that the elements of the matrix $\tilde{\chi}$ have the following general expressions:

$$
\begin{aligned}
\tilde{\chi}_{11} &= [\tilde{b}_2 - (c_2^* e^{i\Phi_C} + c_2 e^{-i\Phi_C})]\eta + [a_2 + \tilde{b}_2 - (b_2^* e^{i\Phi_B} + b_2 e^{-i\Phi_B})]\beta\\
\tilde{\chi}_{12} &= (c_2 e^{-i\Phi_C} - \tilde{b}_2) - \eta[a_2 + \tilde{b}_2 - (b_2^* e^{i\Phi_B} + b_2 e^{-i\Phi_B})]\beta]\\
\tilde{\chi}_{13} &= (a_2 - b_2 e^{-i\Phi_B})\eta\sqrt{\beta}\\
\tilde{\chi}_{22} &= \tilde{b}_2\eta + [a_2 + \tilde{b}_2 - (b_2^* e^{i\Phi_B} + b_2 e^{-i\Phi_B})]\beta\\
\tilde{\chi}_{23} &= (b_2 e^{-i\Phi_B} - a_2)\sqrt{\beta}\\
\tilde{\chi}_{33} &= a_2 && (15)
\end{aligned}
$$

It is also relevant to point out the following:

- When the phases Φ_B and Φ_C vanish, $\beta = 1$ and one takes the 6-texture limit $(\tilde{B}_2 \to 0, i.e.\, \tilde{b} \to 0 \Rightarrow \eta = -1)$, Eq. (14) reduces to

$$\left(\tilde{Y}_2^l\right)_{11} = (2c_2 + a_2 - 2b_2)\, m_1/v$$

$$
\begin{aligned}
\left(\tilde{Y}_2^l\right)_{12} &= (c_2 + a_2 - 2b_2)\,\sqrt{m_1 m_2}/v \\
\left(\tilde{Y}_2^l\right)_{13} &= (b_2 - a_2)\,\sqrt{m_1 m_3}/v \\
\left(\tilde{Y}_2^l\right)_{22} &= (a_2 - 2b_2)\,m_2/v \\
\left(\tilde{Y}_2^l\right)_{23} &= (b_2 - a_2)\,\sqrt{m_2 m_3}/v \\
\left(\tilde{Y}_2^l\right)_{33} &= a_2\,m_3/v
\end{aligned}
\tag{16}
$$

which correspond to the *Ansatz* of Cheng-Sher (See Eq. (32) in Ref. [15]).

- On the other hand, when the phases Φ_B and Φ_C vanish, $\beta = m_2/m_3$ and $\eta = 1$, Eq. (14) reduces to

$$
\begin{aligned}
\left(\tilde{Y}_2^l\right)_{11} &= (\tilde{b}_2 - 2c_2)\,m_1/v \\
\left(\tilde{Y}_2^l\right)_{12} &= (c_2 - \tilde{b}_2)\,\sqrt{m_1 m_2}/v \\
\left(\tilde{Y}_2^l\right)_{13} &= (a_2 - b_2)\,\sqrt{m_1 m_2}/v \\
\left(\tilde{Y}_2^l\right)_{22} &= \tilde{b}_2\,m_2/v \\
\left(\tilde{Y}_2^l\right)_{23} &= (b_2 - a_2)\,m_2/v \\
\left(\tilde{Y}_2^l\right)_{33} &= a_2\,m_3/v
\end{aligned}
\tag{17}
$$

in this case one reproduces the results given in Ref.[20] (See Eq. (24)).

While the diagonal elements $\tilde{\chi}_{ii}$ are real, we notice (Eqs. 15) the appearance of the phases in the off-diagonal elements, which are essentially unconstrained by present low-energy phenomena. As we will see next, these phases modify the pattern of flavor violation in the Higgs sector. For instance, while the Cheng-Sher *Ansatz* predicts that the LFV couplings $(\tilde{Y}_2^l)_{13}$ and $(\tilde{Y}_2^l)_{23}$ vanish when $a_2 = b_2$, in our case this is no longer valid for $\cos\Phi_B \neq 1$. Furthermore the LFV couplings satisfy several relations, such as: $|\tilde{\chi}_{23}| = |\tilde{\chi}_{13}|$, which simplifies the parameter freedom.

Finally, in order to perform our phenomenological study we find convenient to rewrite the lagrangian given in Eq. (4) in terms of the $\tilde{\chi}_{ij}$'s as follows:

$$
\begin{aligned}
\mathcal{L}_Y^l &= \frac{g}{2}\,\bar{l}_i \left[\left(\frac{m_i}{m_W}\right)\frac{\cos\alpha}{\cos\beta}\,\delta_{ij} + \frac{\sin(\alpha-\beta)}{\sqrt{2}\,\cos\beta}\left(\frac{\sqrt{m_i m_j}}{m_W}\right)\tilde{\chi}_{ij}\right] l_j H^0 \\
&+\frac{g}{2}\,\bar{l}_i \left[-\left(\frac{m_i}{m_W}\right)\frac{\sin\alpha}{\cos\beta}\,\delta_{ij} + \frac{\cos(\alpha-\beta)}{\sqrt{2}\,\cos\beta}\left(\frac{\sqrt{m_i m_j}}{m_W}\right)\tilde{\chi}_{ij}\right] l_j h^0 \\
&+\frac{ig}{2}\,\bar{l}_i \left[-\left(\frac{m_i}{m_W}\right)\tan\beta\,\delta_{ij} + \frac{1}{\sqrt{2}\,\cos\beta}\left(\frac{\sqrt{m_i m_j}}{m_W}\right)\tilde{\chi}_{ij}\right] \gamma^5 l_j A^{0.}
\end{aligned}
\tag{18}
$$

where, unlike the Cheng-Sher *Ansatz*, $\tilde{\chi}_{ij}$ $(i \neq j)$ are complex.

3 Bounds on the LFV Higgs parameters

Constrains on the LFV-Higgs interaction will be obtained by studying LFV transitions, which include the 3-body modes $(l_i \to l_j l_k \bar{l}_k)$, radiative decays $(l_i \to l_j + \gamma)$, $\mu - e$ conversion in nuclei, as well as the (LFC) muon anomalous magnetic moment.

3.1 *LFV three-body decays.* To evaluate the LFV leptonic couplings, we calculate the decays $l_i \to l_j l_k \bar{l}_k$, including the contribution from the three Higgs bosons (h^0, H^0 and A^0).We obtain the following expression for the branching ratio:

$$\begin{aligned} Br(l_i \to l_j l_k \bar{l}_k) &= \frac{5\,\delta_{jk}+2}{3}\,\frac{\tau_i}{2^{11}\,\pi^3}\,\frac{m_j\,m_k^2\,m_i^6}{v^4}\left\{\frac{\cos^2(\alpha-\beta)\,\sin^2\alpha}{m_{h^0}^4}+\frac{\sin^2(\alpha-\beta)\,\cos^2\alpha}{m_{H^0}^4}\right. \\ &\left. -2\,\frac{\cos(\alpha-\beta)\,\sin(\alpha-\beta)\,\cos\alpha\,\sin\alpha}{m_{h^0}^2\,m_{H^0}^2}+\frac{\sin^2\beta}{m_{A^0}^4}\right\}\frac{\chi_{ij}^2}{2\,\cos^4\beta} \qquad (19) \end{aligned}$$

where τ_i denotes the life time of the lepton l_i and we have assumed $\chi_{kk} \ll 1$; this result agrees with Ref. [20].

In particular, for the decay $\tau^- \to \mu^-\mu^+\mu^-$ we obtain the following expression for the branching ratio:

$$\begin{aligned} Br(\tau^- \to \mu^-\mu^+\mu^-) &= \frac{5}{3}\,\frac{\tau_\tau}{2^{12}\,\pi^3}\,\frac{m_2^3\,m_3^6}{v^4}\left\{\frac{\cos^2(\alpha-\beta)\,\sin^2\alpha}{m_{h^0}^4}+\frac{\sin^2(\alpha-\beta)\,\cos^2\alpha}{m_{H^0}^4}\right. \\ &\left. -2\,\frac{\cos(\alpha-\beta)\,\sin(\alpha-\beta)\,\cos\alpha\,\sin\alpha}{m_{h^0}^2\,m_{H^0}^2}+\frac{\sin^2\beta}{m_{A^0}^4}\right\}\frac{\chi_{23}^2}{\cos^4\beta} \qquad (20) \end{aligned}$$

here τ_τ corresponds to the life time of the τ lepton (we have also assumed $\chi_{22} \ll 1$).

Using the experimental result $Br(\tau^- \to \mu^-\mu^+\mu^-) < 1.9 \times 10^{-6}$, we get an upper bound on χ_{23} ($(\chi_{23})_{u.b.}^{\tau\to 3\mu}$) as a function of α and $\tan\beta$. In Fig. 1 we show the value of this bound as a function of $\tan\beta$ for $\alpha = \beta - \pi/4$, $\alpha = \beta - \pi/3$ and $\alpha = \beta - \pi/2$, taking $m_{h^0} = 115\ GeV$ and $m_{H^0} = m_{A^0} = 300\ GeV$.

Taking $\chi_{23} \approx 1$, $\tan\beta \approx 30$ and $\pi/4 < \beta - \alpha < \pi/2$, in Eq. (20) one finds tipically that $Br(\tau \to 3\mu) \sim 10^{-8}$, which puts it into the regime that is experimentally accessible at τ-factories over the next few years. At LHC and SuperKEKB, limits in the range of 10^{-9} should be achievable [21], allowing a deeper probe into the parameter space.

3.2 *Radiative decays.* The branching ratio of $\mu^+ \to e^+\gamma$ at one loop level is given by [22]

$$Br(\mu^+ \to e^+\gamma) = \frac{\alpha_{em}\tau_\mu m_1 m_2^4 m_3^4}{2^{12}\pi^4 v^4\cos^4\beta}\,\chi_{23}^2\,\chi_{13}^2\left\{\frac{\cos^4(\alpha-\beta)}{m_{h^0}^4}\left|\ln\frac{m_3^2}{m_{h^0}^2}+\frac{3}{2}\right|^2\right.$$

$$+2\frac{\cos^2(\alpha-\beta)\sin^2(\alpha-\beta)}{m_{h^0}^2 m_{H^0}^2}\left|\ln\frac{m_3^2}{m_{h^0}^2}+\frac{3}{2}\right|\left|\ln\frac{m_3^2}{m_{H^0}^2}+\frac{3}{2}\right|$$
$$\left.+\frac{\sin^4(\alpha-\beta)}{m_{H^0}^4}\left|\ln\frac{m_3^2}{m_{H^0}^2}+\frac{3}{2}\right|^2+\frac{1}{m_{A^0}^4}\left|\ln\frac{m_3^2}{m_{A^0}^2}+\frac{3}{2}\right|^2\right\} \qquad (21)$$

From Eqs. (15) we have $\chi_{23} = \chi_{13} = |(a_2 - b_2 e^{-i\Phi_B})|\sqrt{\beta}$. We will make use of the current experimental upper bound $Br(\mu^+ \to e^+\gamma) < 1.2 \times 10^{-11}$ [23] to constraint $\chi_{23}(\chi_{13})$ as a function of α and $\tan\beta$. Assuming $m_{h^0} = 115\ GeV$ and $m_{H^0} = m_{A^0} = 300\ GeV$, we depict in Fig. 2 the value of the upper bound on χ_{23} $((\chi_{23})_{u.b.}^{\mu\to e\gamma})$ as a function of $\tan\beta$, again for $\alpha = \beta - \pi/4$, $\alpha = \beta - \pi/3$ and $\alpha = \beta - \pi/2$. A new experiment at PSI will measure the process $\mu^+ \to e^+\gamma$ with a sensitivity of 1 event for $Br(\mu^+ \to e^+\gamma) = 10^{-14}$ [24], which would improve the upper bound on χ_{23} by a factor $\sim 10^{-3/4} \approx 0.18$.

3.3 *$\mu - e$ conversion.* The formulas of the conversion branching ratios for the lepton flavor violating muon electron process in nuclei at large $\tan\beta$, in the aluminum and lead targets, are approximately given by

$$Br(\mu^- Al \to e^- Al) \simeq 1.8 \times 10^{-4}\,\frac{m_1\, m_2^6\, m_p^2\, \tan^6\beta\, \cos^2\beta}{2\, v^4\, m_{H^0}^4\, \omega_{capt}}\,\chi_{12}^2 \qquad (22)$$

and

$$Br(\mu^- Pb \to e^- Pb) \simeq 2.5 \times 10^{-3}\,\frac{m_1\, m_2^6\, m_p^2\, \tan^6\beta\, \cos^2\beta}{2\, v^4\, m_{H^0}^4\, \omega_{capt}}\,\chi_{12}^2, \qquad (23)$$

respectively, where ω_{capt} is the rate for muon capture in the nuclei [25]. The values are $\omega_{capt} = 0.7054 \times 10^6\, s^{-1}$ and $\omega_{capt} = 13.45 \times 10^6\, s^{-1}$ in the aluminum and the lead nuclei, respectively [26]. There are several planned experiments which are aiming at improving the bounds of the branching fractions for relevant processes by three or four orders of magnitude [27, 28, 29]. In particular, the MECO experiment will search for the coherent conversion of muons to electrons in the field of a nucleus with a sensitivity of 1 event for 5×10^{16} muon captures, *i. e.* $Br(\mu^- \mathcal{N} \to e^- \mathcal{N}) < 2 \times 10^{-17}$ [30, 31]. Taking $m_{H^0} = 300$ GeV, we plot in Fig. 3 the value of the upper bound on χ_{12} $((\chi_{12})_{u.b.}^{\mu\mathcal{N}\to e\mathcal{N}})$ as a function of $\tan\beta$ for Al and Pb, for the current experimental measurement $Br(\mu^- \mathcal{N} \to e^- \mathcal{N}) < 6.1 \times 10^{-13}$ [31]. In Fig. 4, we show the same as in Fig. 3 but for $Br(\mu^- \mathcal{N} \to e^- \mathcal{N}) < 2 \times 10^{-17}$.

3.4 *Muon anomalous magnetic moment.* Taking the average value of the measurements of the muon $(g-2)$ from [33] and the recent analysis by different groups [34, 35] one can conlude that

$$\Delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} \approx 300 \pm 100 \times 10^{-11}. \qquad (24)$$

The contribution to the muon $g - 2$ of the one loop level flavor changing diagram is given as follows.

$$\Delta a_\mu = \pm \frac{1}{16\pi^2} \frac{m_2^2\, m_3^2}{v^2\, m_{\phi^0}^2} \frac{\cos^2(\alpha-\beta)}{\cos^2\beta} \left(\ln \frac{m_{\phi^0}^2}{m_3^2} - \frac{3}{2} \right) \chi_{23}^2 \tag{25}$$

where the sign +(-) is for scalar $\phi^0 = h^0$ (pseudoscalar, $\phi^0 = A^0$) exchanges [32, 36, 37, 38]. From Eq. (24) is clear that we need to increase the theoretical value of a_μ. Hence, we will consider the contribution of h^0 to the muon $(g-2)$ assuming $\chi_{23} = 1$, namely $\Delta a_\mu^{h^0}(\chi_{23} = 1)$. We take $m_{h^0} = 115\ GeV$ and present in Fig. 5 the result for such contribution as a function of $\tan\beta$ for $\alpha = \beta - \pi/4$ and $\alpha = \beta - \pi/3$. We observe that $\Delta a_\mu^{h^0}(\chi_{23} = 1) < 240 \times 10^{-11}$. On the other hand, the contribution to the anomalous magnetic moment from two-loop double scalar-exchanging diagrams is comparable with the one from the corresponding flavor-changing one loop diagrams [32]. It was already shown in Ref. [32] that the two-loop double scalar (pseudoscalar) exchanging diagrams give negative (positive) contributions, which have opposite signs as the one from one loop scalar (pseudoscalar) exchanging diagram. Hence, we can conclude that it would be very hard to constrain χ_{23} from the muon $g-2$ measurements, or to explain such deviation from the pure Higgs sector in case the signal is confirmed.

Thus, we conclude from this section that the bounds on the LFV parameters are given as follows.

- $\chi_{12} < 5 \times 10^{-1}$, from $\mu^- - e^-$ conversion experiments.
- $\chi_{13} = \chi_{23} < 6 \times 10^{-1}$, from the radiative decay $\mu^+ \to e^+\gamma$ measurements.

However, one can still say that at the present time the couplings χ_{ij}'s are not highly constrained, thus they could induce interesting direct LFV Higgs signals at future colliders.

4 Probing the LFV Higgs couplings at future colliders

In order to probe the LFV Higgs vertices we shall consider both the search for the LFV Higgs decays at future hadron colliders (LHC mainly), as well as the production of Higgs bosons in the collisions of electrons and muons, which was proposed some time ago [39], namely we shall evaluate the reaction $e\mu \to h^0 \to \tau\tau$.

4.1 *Search for LFV Higgs decays at Hadron colliders.* We shall concentrate here on the LFV Higgs decays $\phi_i \to \tau\mu$, which has a very small branching ratio within the context of the SM with light neutrinos ($\leq 10^{-7} - 10^{-8}$), so that this channel becomes an excellent window for probing new physics [10, 40, 41]. The decay width for the procces $\phi_i \to \tau\mu$ (adding both final states $\tau^+\mu^-$ and $\tau^-\mu^+$) can be written in terms of the decay width $\Gamma(H_i \to \tau\tau)$, as follows:

$$\Gamma(\phi_i \to \tau\mu) \;=\; (R^{\phi}_{\tau\mu})^2\, \Gamma(H_i \to \tau\tau) \tag{26}$$

where

$$R^{\phi}_{\tau\mu} = \frac{g_{\phi\tau\mu}}{g_{\phi\tau\tau}} \cong \frac{\sin(\alpha-\beta)}{\cos\alpha}\sqrt{\frac{m_\mu}{m_\tau}}\,\tilde{\chi}_{23} \tag{27}$$

Therefore, the Higgs branching ration can be approximated as: $Br(\phi_i \to \tau\mu) = (R^{\phi}_{\tau\mu})^2 \times Br(\phi_i \to \tau\tau)$. We calculated the branching fraction for $h \to \tau\mu$, and find that it reaches values of order 10^{-2} in the THDM-III; for comparison, we notice that in the MSSM case, even for large values of $\tan\beta$, one only gets $Br(h \to \tau\mu) \simeq 10^{-4}$.

These values of the branching ratio enter into the domain of detectability at hadron colliders (LHC), provided that the cross-section for Higgs production were of order of the SM one. Large values of $\tan\beta$ are also associated with large b-quark Yukawa coupling, which in turn can produce and enhancement on the Higgs production cross-sections at hadron colliders, even for the heavier states H^0 and A^0 either by gluon fusion or in the associated production of the Higgs with b-quark pairs; some values are shown in table 1; these were obtained using HIGLU [42]. Thus, even the heavy Higgs bosons of the model could be detected through this LFV mode.

$m_{H,A}$ [GeV]	σ^H_{gg} [pb]	σ^A_{gg} [pb]	σ^H_{bb} [pb] ($\simeq \sigma^A_{bb}$)
150	126.4 (492.6)	129.1 (525.)	200 (800)
200	29.5 (114.3)	29.1 (120.)	100 (400)
300	3.6 (13.5)	3.15 (13.6)	20 (80)
350	1.6 (5.9)	1.2 (5.6)	12 (48)
400	0.75 (2.75)	0.73 (2.8)	8 (32)

Table 1. Cross-section for Higgs production at LHC, through gluon fusion ($\sigma^{H,A}_{gg}$) and in association with $b\bar{b}$ quarks, ($\sigma^{H,A}_{bb}$), for $\tan\beta = 30$ (60).

For instance, for $m_{H,A} = 150$ GeV and $\tan\beta = 30(60)$ the cross-section through gluon fusion at LHC is about 126.4 (492.6) pb [42], then with $Br(H \to \tau\mu) \simeq 10^{-2}(10^{-3})$ and an integrated luminosity of $10^5\,pb^{-1}$, LHC can produce about $10^5(10^4)$ LFV Higgs events. In Ref. [43] it was proposed a series of cuts to reconstruct the hadronic and electronic tau decays from $h \to \tau\mu$ and separate the signal from the backgrounds, which are dominated by Drell-Yan tau pair and WW pair production. According to these studies [43], even SM-like cross sections and $m_\phi \simeq 150\,GeV$, one coud detect at LHC the LFV Higgs decays with a branching ratio of order 8×10^{-4}, which means that our signal is clearly detectable.

4.2 *Tests of LFV Higgs couplings at eμ-colliders.* Another option to search for LFV Higgs couplings, but now involving the electron-muon-Higgs couplings, would be to search for the reaction: $e^-(p_a) + \mu^+(p_b) \to h^0 \to \tau^-(p_c) + \tau^+(p_d)$. Assuming $\chi_{33} \ll 1$, the result for the cross section is given by:

$$\sigma(e^-\mu^+ \to \tau^-\tau^+) \;=\; \frac{s\,m_1\,m_2\,m_3^2}{32\,\pi\,v^4\,\cos^4\beta}\,\chi^2_{12}$$

$$
\begin{aligned}
&\{|D_{h^0}(s)|^2 \cos^2(\alpha-\beta)\, \sin^2\alpha \\
&-2Re\{D_{h^0}(s)\, D^{\star}_{H^0}(s)\}\, \cos(\alpha-\beta)\, \sin(\alpha-\beta)\, \sin\alpha\, \cos\alpha \\
&+|D_{H^0}(s)|^2 \sin^2(\alpha-\beta)\, \cos^2\alpha + |D_{A^0}(s)|^2\, \sin^2\beta\},
\end{aligned} \tag{28}
$$

where $D_{\phi^0}(s)$ denotes the Breit-Wigner form of the ϕ^0 propagator

$$
D_{\phi^0}(s) = (s - m_{\phi^0}^2 + i m_{\phi^0}\, \Gamma_{tot}^{\phi^0})^{-1} \tag{29}
$$

and $s = (p_a + p_b)^2 = (p_c + p_d)^2$.

The non-observation of at least an event in a year would imply that

$$
\sigma(e^-\mu^+ \to \tau^-\tau^+) \times \text{luminosity} \times 1\,\text{year} < 1, \tag{30}
$$

which would allow us to put an upper bound on χ_{12}, namely $(\chi_{12})^{e\mu\to\tau\tau}_{u.b.}(s)$ as a function of α and $\tan\beta$. In order to obtain numerical results, we take $\Gamma^{h^0}_{tot} = 0.004\, GeV$ for $m_{h^0} = 115\, GeV$; $\Gamma^{H^0}_{tot} = 0.14\, GeV$ for $m_{H^0} = 300\, GeV$; and $\Gamma^{A^0}_{tot} = 0.045\, GeV$ for $m_{A^0} = 300\, GeV$ [44] and a luminosity $\mathcal{L} = 2\times 10^{32}\, cm^{-2}\, s^{-1}$ [39]. We present our numerical results for $(\chi_{12})^{e\mu\to\tau\tau}_{u.b.}(s = m^2_{h^0})$ and $(\chi_{12})^{e\mu\to\tau\tau}_{u.b.}(s = m^2_{H^0} = m^2_{A^0})$ in Fig. 6 and Fig. 7, respectively.

We can also estimate the number of events $N^{e\mu\to\tau\tau}(s)$ by taking for χ_{12} the value for the current upper bound on χ_{12} obtained from the measurements in $\mu^- - e^-$ conversion experiments, namely $\chi_{12} = (\chi_{12})^{\mu\mathcal{N}\to e\mathcal{N}}_{u.b.}$ as a function of $\tan\beta$ for Al, for the current experimental measurement $Br(\mu^-\mathcal{N} \to e^-\mathcal{N}) < 6.1\times 10^{-13}$ [31]. Hence, we get

$$
N^{e\mu\to\tau\tau}(s) = \sigma(e^-\mu^+ \to \tau^-\tau^+) \times \text{luminosity} \times 1\,\text{year}, \tag{31}
$$

as a function of α and $\tan\beta$. In order to obtain numerical results, we take $\Gamma^{h^0}_{tot} = 0.004\, GeV$ for $m_{h^0} = 115\, GeV$; $\Gamma^{H^0}_{tot} = 0.14\, GeV$ for $m_{H^0} = 300\, GeV$; and $\Gamma^{A^0}_{tot} = 0.045\, GeV$ for $m_{A^0} = 300\, GeV$ [44] and a luminosity $\mathcal{L} = 2\times 10^{32}\, cm^{-2}\, s^{-1}$ [39]. We present our numerical results for $N^{e\mu\to\tau\tau}(s = m^2_{h^0})$ and $N^{e\mu\to\tau\tau}(s = m^2_{H^0} = m^2_{A^0})$ in Fig. 8 and Fig. 9, respectively. We obtain around 100 events *per* year, which is very likely detectable.

5 Conclusions

We have studied in this paper the lepton-Higgs couplings that arise in the THDM-III, using a Hermitic four-texture form for the leptonic Yukawa matrix. Because of this, although the fermion-Higgs couplings are complex, the CP-properties of h^0, H^0 (even) and A^0 (odd) remmain valid.

We have derived bounds on the LFV parameters of the model, using current experimental bounds on LFV transitions. Our resulting bounds can be summarized as follows.

- $\chi_{12} < 5\times 10^{-1}$, from $\mu^- - e^-$ conversion experiments.
- $\chi_{13} = \chi_{23} < 6\times 10^{-1}$, from the radiative decay $\mu^+ \to e^+\gamma$ measurements.

However, one can say that the present bounds on the couplings χ_{ij}'s still allow the possibility to study interesting direct LFV Higgs signals at future colliders.

In particular, the LFV couplings of the neutral Higgs bosons, can lead to new discovery signatures of the Higgs boson itself. For instance, the branching fraction for $H/A \to \tau\mu$ can be as large as 10^{-2}, while $Br(h \to \tau\mu)$ is also about 10^{-2}. These LFV Higgs modes complement the modes $B^0 \to \mu\mu$, $\tau \to 3\mu$, $\tau \to \mu\gamma$ and $\mu \to e\gamma$, as probes of flavor violation in the THDM-III, which could provide key insights into the form of the Yukawa mass matrix.

On the other hand, one can also relate our results with the SUSY-induced THDM-III, by considering the effective Lagrangian for the couplings of the charged leptons to the neutral Higgs fields, namely:

$$-\mathcal{L} = \overline{L}_L Y_l l_R \phi_1^0 + \overline{L}_L Y_l \left(\epsilon_1 \mathbf{1} + \epsilon_2 Y_\nu^\dagger Y_\nu\right) l_R \phi_2^{0*} + h.c. \qquad (32)$$

In this language, LFV results from our inability to simultaneously diagonalize the term Y_l and the non-holomorphic loop corrections, $\epsilon_2 Y_l Y_\nu^\dagger Y_\nu$. Thus, since the charged lepton masses cannot be diagonalized in the same basis as their Higgs couplings, this will allow neutral Higgs bosons to mediate LFV processes with rates proportional to ϵ_2^2. In terms of our previous notation we have: $\tilde{Y}_2 = \epsilon_2 Y_l {Y_\nu}^\dagger Y_\nu$. An study of the values for ϵ_2 resulting from general soft breaking terms in the MSSM is underway.

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Figure Captions

Fig. 1: The upper bound $(\chi_{23})^{\tau\to 3\mu}_{u.b.}$ as a function of $\tan\beta$ for $\alpha = \beta - \pi/4$, $\alpha = \beta - \pi/3$, $\alpha = \beta - \pi/2$, with $Br(\tau^- \to \mu^-\mu^+\mu^-) < 1.9 \times 10^{-6}$, taking $m_{h^0} = 115\ GeV$ and $m_{H^0} = m_{A^0} = 300\ GeV$.

Fig. 2: The upper bound $(\chi_{23})^{\mu\to e\gamma}_{u.b.}$ as a function of $\tan\beta$ for $\alpha = \beta - \pi/4$, $\alpha = \beta - \pi/3$, $\alpha = \beta - \pi/2$, with $Br(\mu^+ \to e^+\gamma) < 1.2 \times 10^{-11}$, taking $m_{h^0} = 115\ GeV$ and $m_{H^0} = m_{A^0} = 300\ GeV$.

Fig. 3: The upper bound $(\chi_{12})^{\mu\mathcal{N}\to e\mathcal{N}}_{u.b.}$ as a function of $\tan\beta$ for Al, Pb, with $Br(\mu^-\mathcal{N} \to e^-\mathcal{N}) < 6.1 \times 10^{-13}$ and assuming $m_{H^0} = 300\ GeV$.

Fig. 4: The same as in Fig. 3, but taking $Br(\mu^-\mathcal{N} \to e^-\mathcal{N}) < 2 \times 10^{-17}$.

Fig. 5: $\Delta a^{h^0}_\mu$ as a function of $\tan\beta$ for $\alpha = \beta - \pi/4$, $\alpha = \beta = -\pi/3$, with $\chi_{23} = 1$ and assuming $m_{h^0} = 115\ GeV$.

Fig. 6: The upper bound $(\chi_{12})^{e\mu\to\tau\tau}_{u.b.}$ for $s = m^2_{h^0} = (115\, GeV)^2$, with $\Gamma^{h^0}_{tot} = 0.004\, GeV$, as a function of $\tan\beta$ for $\alpha = \beta-\pi/4$, $\alpha = \beta-\pi/3$, when $\sigma(e^-\mu^+ \to \tau^-\tau^+) \times$luminosity$\times$ 1 year < 1, taking $\mathcal{L} = 2 \times 10^{32}\, cm^{-2}\, s^{-1}$

Fig. 7: The upper bound $(\chi_{12})^{e\mu\to\tau\tau}_{u.b.}$ for $s = m^2_{H^0} = m^2_{A^0} = (300\, GeV)^2$, with $\Gamma^{H^0}_{tot} = 0.14\, GeV$ and $\Gamma^{A^0}_{tot} = 0.045\, GeV$, as a function of $\tan\beta$ for $\alpha = \beta-\pi/4$, $\alpha = \beta-\pi/3$, $\alpha = \beta-\pi/2$, when $\sigma(e^-\mu^+ \to \tau^-\tau^+) \times$ luminosity $\times$ 1 year < 1, taking $\mathcal{L} = 2 \times 10^{32}\, cm^{-2}\, s^{-1}$

Fig. 8: Number of events $N^{e\mu\to\tau\tau}$ for $s = m^2_{h^0} = (115\, GeV)^2$, taking $\chi_{12} = (\chi_{12})^{\mu Al\to eAl}_{u.b.}$ for $Br(\mu^- Al \to e^- Al) < 6.1 \times 10^{-13}$ with $\Gamma^{h^0}_{tot} = 0.004\, GeV$, as a function of $\tan\beta$ for $\alpha = \beta - \pi/4$, $\alpha = \beta - \pi/3$, taking $\mathcal{L} = 2 \times 10^{32}\, cm^{-2}\, s^{-1}$

Fig. 9: Number of events $N^{e\mu\to\tau\tau}$ for $s = m^2_{H^0} = m^2_{A^0} = (300\, GeV)^2$, taking $\chi_{12} = (\chi_{12})^{\mu Al\to eAl}_{u.b.}$ for $Br(\mu^- Al \to e^- Al) < 6.1 \times 10^{-13}$ with $\Gamma^{H^0}_{tot} = 0.14\, GeV$ and $\Gamma^{A^0}_{tot} = 0.045\, GeV$, as a function of $\tan\beta$ for $\alpha = \beta - \pi/4$, $\alpha = \beta - \pi/3$, $\alpha = \beta - \pi/2$, taking $\mathcal{L} = 2 \times 10^{32}\, cm^{-2}\, s^{-1}$

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